Selection of an Optimal Cost Index for Airline Hub Operation

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The introduction of time-based control in an onboard system creates new opportunities to meet the requirements and goals of the airlines. This paper presents a method for obtaining the optimum cost index for a given flight, with a given flight route and a scheduled time of arrival, where the cost function takes into account the cost of predicted arrival time errors. A 1000 n.mi. route is chosen and the optimal cost index is determined for different wind conditions and schedule policies. The policy speed is allowed to vary between maximum-range cruise speed and Mach 0.8. The effects of crew overtime, hubbing, and passenger satisfaction costs on the selection of an optimal cost index are analyzed. It is assumed that the takeoff takes place either at the scheduled takeoff time or later, so that the expiration of the scheduled flight time coincides with the scheduled landing time or is a little behind. Finally, the effect of the delayed takeoff on the optimal cost index is delineated.

Nomenclature

C_f = cost of fuel
C_t^* = baseline cost of time
CI = cost index
D = drag
E = specific energy
f = fuel flow rate
H = Hamiltonian
h_{cr} = cruise altitude
J = cost function
m = mass of aircraft
T = thrust
t_f = terminal time
t_i = initial time
$T_{\max}$ = maximum thrust
$T_{\min}$ = minimum thrust
T_0 = climb/descent thrust
V = airspeed
V_{cr} = cruise airspeed
V_0 = climb/descent speed
V_w = wind speed
x = range
γ = air mass flight path angle
ϵ = singular perturbation parameter
λ_E = energy adjoint variable
λ_m = mass adjoint variable
λ_x = range adjoint variable
$\phi(t_f)$ = overtime, hubbing, and passenger satisfaction costs

Introduction

THE designer of a flight management system (FMS) tries to enable the airlines to fly their airplanes in a cost-effective manner. The most obvious costs related to a flight are those of fuel and direct operations associated with maintenance and crew. If a flight arrives late, there are other costs such as crew overtime and those associated with lost revenue due to missed connecting flights and potential losses due to customer dissatisfaction.

The concept of long-range cruise (LRC) is an attempt to choose a speed that reasonably balances the concerns of fuel economy against those of flight-hour related costs and speed stability. LRC was initially supplied by the manufacturers to the airlines as a recommended cruise speed. Recent improvements in the capabilities of onboard flight management systems (FMS) have allowed more sophisticated speed control. The concept of a "cost index" has been implemented in some of these systems. This is a number that is the ratio of the time-related costs (expressed in dollars per hour) to fuel costs (in cents per pound). The fuel and time-related costs must be determined in order to calculate a value for the cost index. The cost of fuel is easy to find, but the cost of flight time is difficult to evaluate. Maintenance costs must be divided into two components: flight-hour costs and cyclic costs. It is the noncyclic or flight-hour maintenance costs that must be considered.

Crew costs are set by contracts and the crew is usually guaranteed a minimum pay plus overtime if the actual flight time exceeds the scheduled flight time. In this case, the cost function is more complicated than a simple cost per hour. It is possible to optimally select a cost index that would minimize the total cost, including direct operations, overtime, and hubbing/passenger satisfaction.

The cost index can be used to give four-dimensional operational capability. As the cost index is changed, the total flight time will vary. As the flight progresses, the cost index will have to be adjusted to account for changes in the terminal time objective.

Airlines have genuine concerns, such as schedule reliability and hubbing, that are reflected in their policies. However, their current operating methods prevent them from meeting their goals in a cost-efficient manner. To use an FMS as intended, an airline analyzes its operating costs and calculates a value for the cost index. But most airlines start with a predetermined policy cruise speed and then find find the cost index that results in the policy speed. The goal is to minimize fuel costs within the constraints of speed restrictions.

Terminal Time Costs

Part of the airlines' criticism of the cost index is that it does not accurately account for their time costs, in particular crew overtime and hubbing. The crew overtime cost is

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restricted to the cabin crew. It starts accumulating at the expiration of the scheduled flight time. The hubbing cost starts a few minutes after the scheduled landing time and accumulates in discrete steps for a period of 20-30 min. The passenger satisfaction cost can be parabolic in nature and will start accumulating at the scheduled landing time. This cost can be associated with hubbing, but may exist even when there are no connecting flights.

As seen in Fig. 1, the overtime cost increases linearly after the expiration of the scheduled flight time. Figure 2 suggests that even the hubbing cost can be approximated by a straight line. The rationale behind the hubbing cost is simple. This cost starts accumulating a few minutes after the scheduled landing time and increases in discrete steps depending on the number of missed connections. The cost finally becomes constant if there are no more connecting flights. In addition, if the actual takeoff occurs at the same time as the scheduled takeoff, then the expiration of the scheduled flight time coincides with the scheduled landing time. Figure 3 depicts a mathematical representation of the passenger satisfaction (goodwill) cost. There is lack of adequate data to define this cost exactly and further research is required in this area.

Mathematical Formulation

For an optimal selection of cost index for airline fleet hub operation, the problem is formulated as follows:

$$\text{Minimize } J = \int_{t_i}^{t_f} (C_f f + C_t^*)\mathrm{d}t + \phi(t_f) \tag{1}$$

subject to

$$\dot{x} = V + V_w \tag{2}$$

$$\dot{m} = -f \tag{3}$$

$$\epsilon \dot{E} = \frac{(V+V_w)(T-D)}{mg} - V_w \gamma \tag{4}$$

ϵ arises after time scaling the equations of motion. The specific energy E is given by

$$E = h + \frac{(V+V_w)^2}{2g} \tag{5}$$

The transversality condition states[1]

$$\left[H + \left(\frac{\partial \phi}{\partial t}\right)_{tf}\right] = 0 \tag{6}$$

where

$$H = C_t^* + C_f f + \lambda_x (V+V_w) - \lambda_m f + \lambda_E \times \left[\frac{(V+V_w)(T-D)}{mg} - V_w \gamma\right] \tag{7}$$

If one looks at the minimum direct operating cost problem in which the Hamiltonian is identically zero and compares it with Eq. (6), it becomes obvious that the real cost of time for our problem is $C_t^* + (\partial\phi/\partial t)_{tf}$ and that the cost index is

$$\mathrm{CI} = \frac{C_t^* + (\partial\phi/\partial t)_{tf}}{C_f} \tag{8}$$

where C_f is in cents per pound and $C_t^* + (\partial\phi/\partial t)_{tf}$ is in dollars per hour. As H does not explicitly depend on time, H is a constant. As t_f is free,

$$H = -\left(\frac{\partial \phi}{\partial t}\right)_{tf} \tag{9}$$

The optimum cruise altitude and speed are obtained by minimizing the cruise cost function

$$h_{cr}, V_{cr} = \arg \min_{h,V} \left[\frac{C_t^* + (\partial\phi/\partial t)_{tf} + C_f f}{V + V_w}\right]_{T=D} \tag{10}$$

In the above equation, it is assumed that $\lambda_m = 0$. In reality, V_w will be a function of both altitude and range. The actual winds over a 1000 n.mi. flight can vary substantially and optimization using Eq. (10) will show this. The optimum climb/descent speed and thrust are obtained by minimizing/maximizing the climb/descent cost function at the current energy level as follows:

$$V_0, T_0 = \arg \begin{matrix} \min\limits_{V,\, T_{max} \geq T > D} \\ \max\limits_{V,\, T_{min} \leq T < D} \end{matrix} \left[\frac{C_t^* + (\partial\phi/\partial t)_{tf} + C_f f + \lambda_x (V+V_w)}{(V+V_w)(T-D)/mg - V_w \gamma}\right]_E \tag{11}$$

λ_x in Eq. (11) is equal to the minimum of the cruise cost function in Eq. (10).[2] Also, Eqs. (10) and (11) have been previously derived by Erzberger[3] without the $(\partial\phi/\partial t)_{tf}$ term.

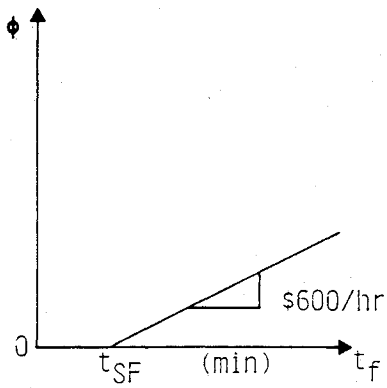

Fig. 1 Overtime cost.

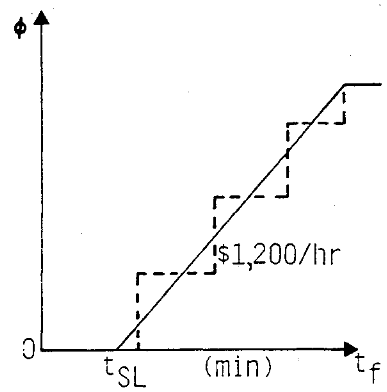

Fig. 2 Hubbing cost.

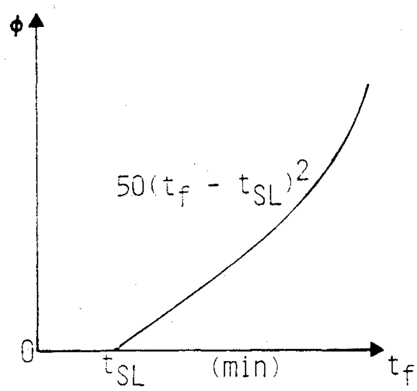

Fig. 3 Goodwill cost.

Method of Solution

As shown by specific examples in Figs. 4 and 5 (for different values of the function ϕ), the optimal cost of time for airline fleet operation is obtained by the intersection point of two curves: 1) the cost of time vs the t_f curve; and 2) the $(\partial\phi/\partial t)_{tf}$ vs t_f curve with its origin coinciding with C_t^* of the cost-of-time axis.

If the intersection point lies on a horizontal segment of $(\partial\phi/\partial t)_{tf}$, it becomes a pure three-dimensional (free t_f) problem and the corresponding cost of time should be used. On the other hand, if the intersection point lies on a vertical segment of $(\partial\phi/\partial t)_{tf}$, it is a pure four-dimensional (fixed t_f) problem and the corresponding t_f should be reached by a cost-of-time iteration. All other cases will be combinations of three- and four-dimensional problems, as shown later.

It is possible to have a negative $(\partial\phi/\partial t)_{tf}$ particularly in a pure four-dimensional problem, where it costs more to arrive early or late (Fig. 5). Given a flight duration, the corresponding cost index can be negative. This is possible only if $(\partial\phi/\partial t)_{tf}$ can be negative. So negative values of $(\partial\phi/\partial t)_{tf}$ are not unrealistic. Also, if the hubbing cost is best represented by a step function, its derivative will be an impulse and the possibility of having a pure four-dimensional solution (where the point of intersection is on a vertical segment) will be much higher.

Results and Discussions

All of the results presented here are for the 767-200 airplane. A 1000 n.mi. route is selected and flight scheduling is performed for standard-day conditions (no wind, standard temperature). The upper half of Fig. 6 plots the fuel consumption vs flight duration for no wind, as well as for head and tail winds (an average magnitude of 5 knots). The lowest point of the curve corresponds to a zero cost index. To the left of this point, the cost index is positive, while to the right it is negative. The lower half of the curve plots the actual cost index against the flight duration, again for different winds. As is evident, for large positive cost indices speed changes vary slowly for large increments in the cost index and for negative cost indices the flight duration changes drastically even for small changes in the cost index. To have a Mach 0.8 schedule in the absence of any wind, one should select a cost index of 95.

Table 1 shows the optimum ATC cruise altitude and the variation in the cruise Mach number for four different time

Table 1 Cruise altitude and Mach for scheduled flight (no wind, range covered = 1000 n.mi.)

Cost of time, $/h	Cruise altitude, ft	Cruise Mach no.
1200	41,000	0.802→0.805
500	41,000	0.792→0.794
0	37,000→41,000	0.750 (37,000 ft)
	(50 n.mi.)	0.780→0.770 (41,000 ft)
−500	37,000	0.704→0.669

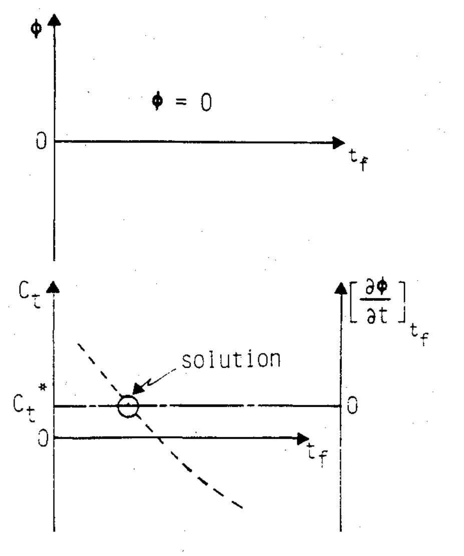

Fig. 4 Typical solution for three-dimensional problem.

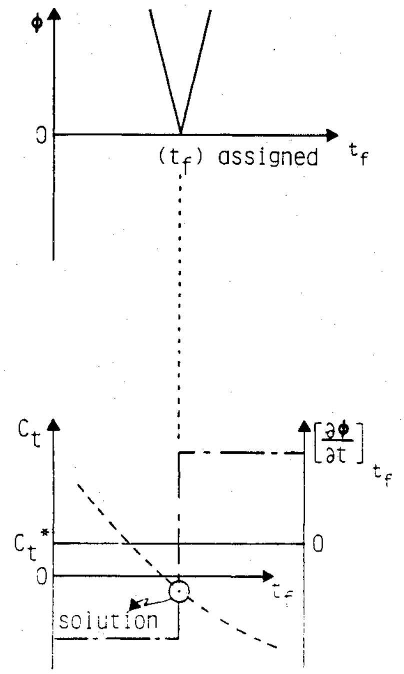

Fig. 5 Typical solution for four-dimensional problem.

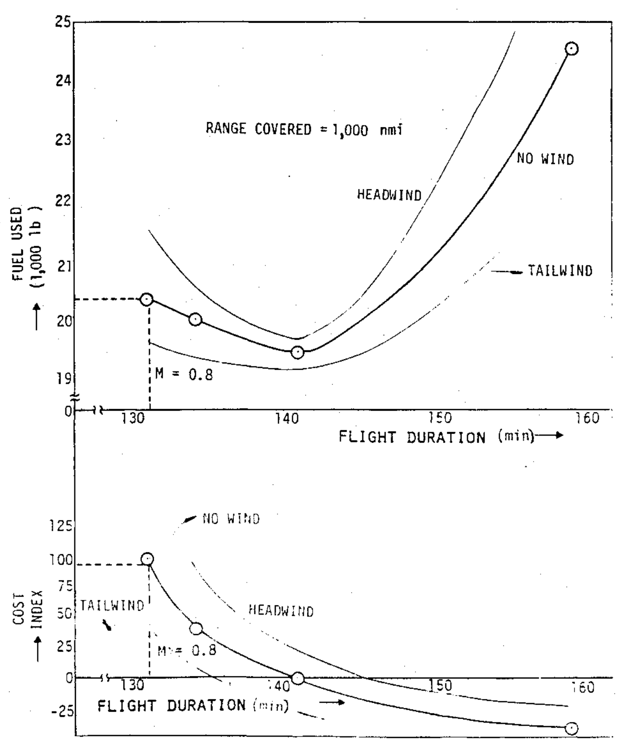

Fig. 6 Flight scheduling.

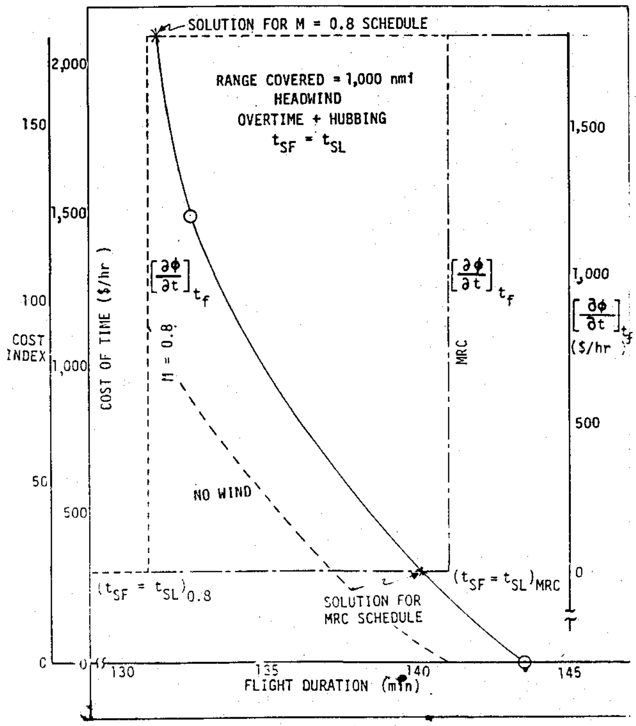

Fig. 7 Flight planning in head wind ($t_{SF} = t_{SL}$).

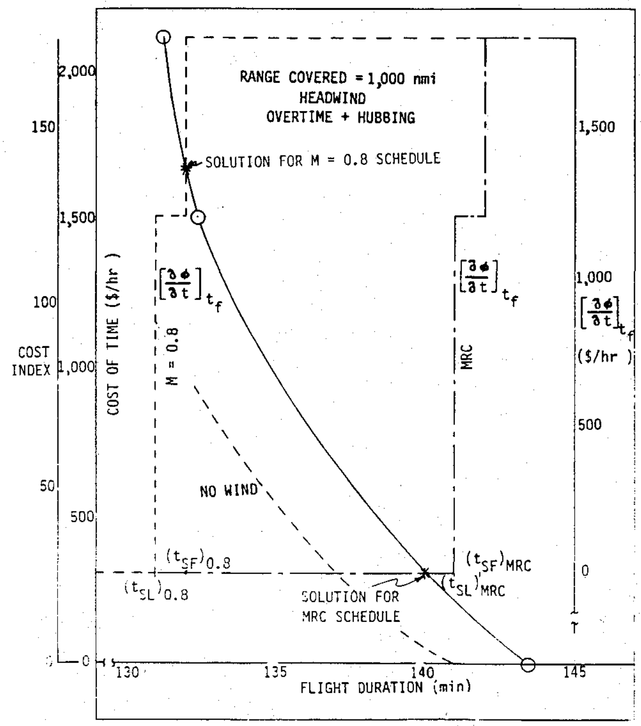

Fig. 8 Flight planning in head wind ($t_{SF} = t_{SL} + 1$ min).

Table 2 Cruise altitude and Mach in head wind (range covered = 1000 n.mi.)

Cost of time, $/h	Cruise altitude, ft	Cruise Mach no.
0	37,000→41,000 (100 n.mi.)	0.752→0.751 (37,000 ft) 0.781→0.770 (41,000 ft)
300	37,000→41,000 (50 n.mi.)	0.758→0.759 (37,000 ft) 0.787→0.789 (41,000 ft)
1500	41,000	0.805→0.809
2100	41,000	0.810→0.815

Table 3 Cruise altitude and Mach in tail wind (range covered = 1000 n.mi.)

Cost of time, $/h	Cruise altitude, ft	Cruise Mach no.
0	37,000→41,000 (50 n.mi.)	0.749 (37,000 ft) 0.779→0.768 (41,000 ft)
300	41,000	0.782→0.785
1500	41,000	0.804→0.808
2100	41,000	0.809→0.814

costs. For a zero cost index only, there is a step climb from 37,000 to 41,000 ft after about 50 n.mi. of cruise. In the other three cases, the airplane likes to cruise at a constant altitude. For positive cost indices, the Mach number has a tendency to increase with time, while for negative cost indices the reverse happens.

Figure 7 illustrates the selection of an optimal cost index for maximum-range cruise (MRC) and Mach 0.8 schedules. The baseline cost of time is taken as $300/h and $C_f = \$0.12$/lb. The preflight planning is done in the presence of a head wind (an average of about 5 knots), while the schedule was for the "no wind" case. It is assumed that the expiration of the scheduled flight time coincides with the scheduled landing time. Only overtime and hubbing costs are considered. The overtime cost is taken as $600/h and the hubbing cost is twice that amount. Goodwill costs are not included in this figure. The dotted line represents $(\partial\phi/\partial t)_{tf}$ for a Mach 0.8 schedule, while the long and short lines show $(\partial\phi/\partial t)_{tf}$ for an MRC schedule. The solid line is the plot of time costs vs flight duration. The points of intersection tell us that, for an MRC schedule, the optimal cost index is 25, while for a Mach 0.8 schedule, a cost index of 175 should be selected.

Table 2 shows the optimal ATC cruise altitude and cruise Mach number variation for four cost indices. For a zero cost index and CI = 25 (the solution for an MRC schedule), a step climb from 37,000 to 41,000 ft takes place, after about 100 n.mi. of cruise for CI = 0 and 50 n.mi. of cruise for CI = 25. In the other two cases, no step climbs are involved. Also, the Mach number tends to increase with time for positive time costs, but for zero time costs the Mach number has a tendency to decrease at a given cruise altitude.

If the airplane takes off a minute after the scheduled flight time will expire a minute after the scheduled landing time, as shown in Fig. 8. So the overtime cost will start a minute after the hubbing cost has started. The points of intersection dictate that for an MRC schedule, the optimal cost index is still 25, while for a Mach 0.8 schedule, one should select a flight duration of 132 min (four-dimensional problem). The resulting cost index will be approximately 140.

Figure 9 shows the results of preflight planning in the presence of a tail wind (average 5 knots). Again, it is assumed that the expiration of the scheduled flight time coincides with the scheduled landing time and the same values as before are used for overtime and hubbing costs. The dotted line represents $(\partial\phi/\partial t)_{tf}$ for a Mach 0.8 schedule, while the long and short lines show $(\partial\phi/\partial t)_{tf}$ for an MRC schedule. The solid line plots the cost of time vs the duration of flight. The points of intersection dictate the selection of a flight duration of 131 min (four-dimensional problem) for a Mach 0.8 schedule, while a cost index of 25 is still optimal for an MRC schedule.

The variation of the cruise Mach number and the optimal ATC cruise altitudes are shown in Table 3. Again, a step climb for a zero cost index takes place after about 50 n.mi.

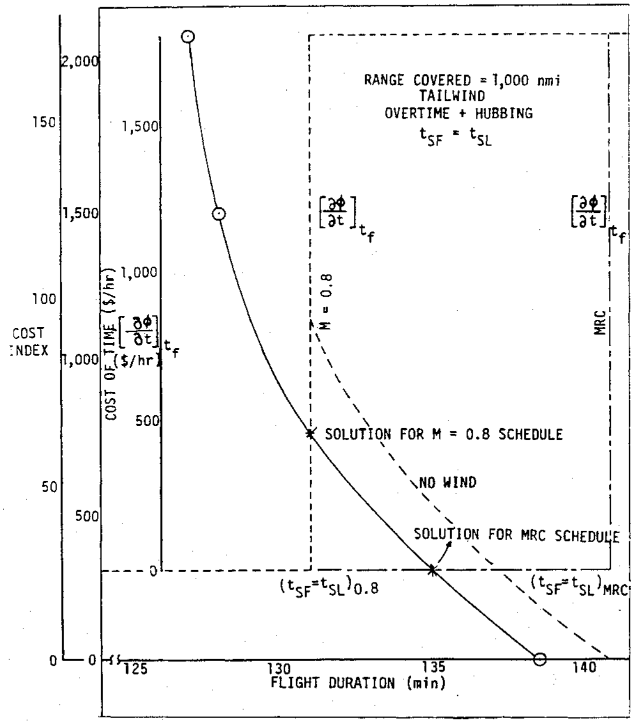

Fig. 9 Flight planning in tail wind ($t_{SF}=t_{SL}$).

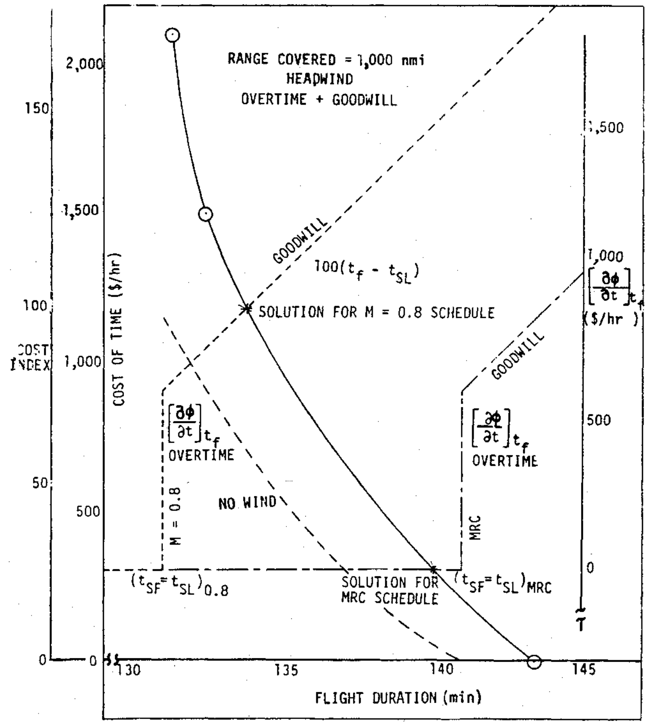

Fig. 10 Flight planning with goodwill and overtime costs.

of cruise. No step climbs are needed for positive costs of time. As before, the Mach number increases with time for positive time costs and decreases when a zero cost index is used.

Finally, Fig. 10 combines overtime and goodwill costs in a preflight plan in the presence of a head wind (average 5 knots). The same overtime cost is used as before and the goodwill cost is given by

$$\phi(t_f) = 50(t_f - t_{SL})^2 \tag{12}$$

where t_f is the terminal time and t_{SL} the scheduled landing time. Again, it is assumed that the expiration of the scheduled flight time coincides with the scheduled landing time. $(\partial\phi/\partial t)_{tf}$ for a Mach 0.8 schedule is shown by the dotted line and the long and short lines represent $(\partial\phi/\partial t)_{tf}$ for an MRC schedule. The solid line plots the cost of time vs the flight duration. The optimal cost index for an MRC schedule is still 25, but a Mach 0.8 schedule should use a value of about 100.

It should be pointed out that it is possible to select the optimal cost index not only for MRC and Mach 0.8, but also for any speed in between. For example, in Fig. 10, if the scheduled flight duration is 133 min, then the dotted curve must be moved to the right (so that its vertical segment is at 133), to find the optimal cost index for this schedule.

Conclusions

An optimal method is proposed for selecting a cost index for airline fleet hub operation. Different terminal time costs, such as overtime, hubbing, and passenger satisfaction are analyzed for wind conditions that are different from the conditions used for scheduling. It is recommended that active research be pursued to define these costs more adequately and also that other costs, such as that of schedule reliability, be included. It is our understanding that such analyses will help the airlines to minimize their overall costs.

References

[1] Bryson, A. E., and Ho, Y. C., *Applied Optimal Control,* Hemisphere Publishing Corp., Washington D.C., 1975, pp. 87-89.

[2] Chakravarty, A., "Four-Dimensional Fuel-Optimal Guidance in the Presence of Winds," *Journal of Guidance, Control and Dynamics,* Vol. 8, Jan.-Feb. 1985, pp. 16-22.

[3] Erzberger, H. and Lee, H., "Constrained Optimum Trajectories with Specified Range," *Journal of Guidance and Control,* Vol. 3, Jan.-Feb. 1980, pp. 78-85.